

First-Ply failure optimization strength of laminated composite pressure vessels

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ABSTRACT

Strengths of laminated composite pressure vessels are studied via both analytical and mathematical approaches. An Axis-symmetric finite element model of graphite/epoxy laminate pressure vessel is established by ANSYS finite element software. Analytical techniques are presented to determine the first-ply failure of laminated composite pressure vessels with different lamination arrangements. A solution algorithm is proposed to investigate the progressive damage and failure properties of composite Pressure vessel with increasing internal pressure. The maximum Principal stress, are validated with analytical and Mathematical for verification of algorithm. The accuracy of the algorithm prediction of first-ply failure strength is verified by the analytical data by ANSYS software.

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Introduction:

The application of composite pressure vessels has drawn close attention in recent years [1-5]. Much work has been devoted to the manufacturing and design aspects of laminated composite pressure vessels [6-10]. In general, the design of laminated composite pressure vessels is achieved by the use of the first-ply failure approach, i.e. a suitable failure criterion is adopted to determine the first-ply failure load, and the classical lamination theory for stress analysis. The suitability of the adopted failure criterion and the classical lamination theory in determining the first-ply failure strength of laminated composite pressure vessels, however, has not been studied in detail nor validated by analytical data. For safety reasons, pressure vessels must be designed for high reliability. A meaningful reliability assessment of a laminated composite pressure vessel relies on the accurate prediction of the first-ply failure strength of the vessel. Therefore, more work must be devoted to the failure analysis of laminated composite pressure vessels if reliable as well as economical vessels are desired. In this paper, first-ply failure optimization of laminated composite pressure vessels is studied via both analytical and mathematical approaches. Mathematical approaches are performed to determine the strengths of laminated composite pressure vessels with different lamination arrangements. And analytical methods commonly used in determining first-ply failure Stress of laminated composite pressure vessels. The suit abilities of different failure criteria is solved by using Mathematical equations in MATLAB. Many studies

have been done for composite laminate optimization. Mustafa Akbulut describes optimization procedure to minimize thickness of laminated composite pressure vessel. The paper shows results of first ply angle optimization for different combinations of in-plane loadings. G Narayana Naik has presented failure mechanism, for maximum stress and Tsai Wu failure criteria. Jacob L. have presented methodology for multi-objective optimization of laminated composite materials which is based on integer coded genetic algorithm.

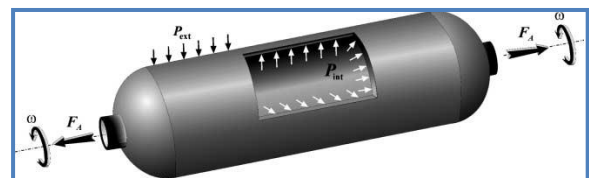


Fig. 1: Mechanical loading on a closed end cylindrical pressure vessel.

Mathematical Formulation for Laminated Structures:

A laminate is made up of perfectly bonded layers of lamina with different fiber orientation to represent an integrated structural component. In most practical applications of composite material, the laminates are considered as thin and loaded along the plane of laminates. A thin orthotropic unidirectional lamina as depicted in Fig.1 has fiber orientation along the 1 direction and the direction transverse to the fiber along

the 2 direction. The x-y coordinates represent the global coordinate system for the lamina.

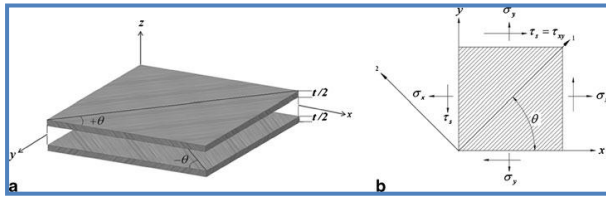


Fig. 2: Anti-symmetric Angle-ply Layers b Stress Components in unidirectional lamina referred to loading and material axes

The pressure vessel is modeled as a symmetrically laminated cylindrical shell of thickness h, length L and radius R, where R refers to the radius of the middle surface. The shell is constructed of an even number of orthotropic layers of equal thickness, t. The fiber orientation θ is defined as the angle between the fiber direction and the longitudinal axis x. The stress resultants in the geometric coordinate axes are given by [10]

$$N = A\varepsilon \text{----- (1)}$$

Where N is the vector of stress resultants, A_{-} is the matrix of extensional stiffness's, ε is the vector of strains. The stress-strain relations for the kth orthotropic layer are given by

$$\{\sigma_{xys}\} = [Q_{xys}] \{\varepsilon_{xys}\} \text{----- (2)}$$

Where σ_{xys} is the vector of stresses for the kth ply, Q_{xys} is the matrix of the transformed material stiffness constants. According to the principle of the strength of materials, the stress resultants of the pressure vessel subjected to internal pressure p are given by

$$N_x = pr/2, N_\phi = Pr, N_x\phi = 0 \text{----- (3)}$$

Where N_x, N_ϕ are stress resultants in the axial and circumferential directions, respectively; $N_x\phi$ is the shear stress resultant which is zero due to the symmetry of the lamination. The first-ply failure analysis of the laminated composite pressure vessel is performed via the use of a suitable failure criterion. Herein, a number of phenomenological failure criteria are adopted in the analysis. For comparison purpose, the laminated composite pressure vessel is also analyzed using the finite element method which is formulated on the basis of the first-order shear deformation theory [11].

Stresses and strains of composite shell:

The Stress Strain Relation of a composite lamina may be written in the following Matrix form where the Q_{ij} are defined in terms of lamina, Young's modulus and Poisson's ratio as follows:

$$Q_{11} = \frac{E_{11}}{1 - \mu_{12}\mu_{21}} \quad Q_{22} = \frac{E_{22}}{1 - \mu_{12}\mu_{21}} \quad Q_{12} = \frac{\mu_{12}E_{11}}{1 - \mu_{12}\mu_{21}}$$

$$Q_{16} = Q_{26} \quad Q_{66} = Q_{12} \quad \mu_{21} = \mu_{12} \frac{E_{22}}{E_{11}}$$

The terms within $[Q_{126}]$ are defined to be

$$Q_{126} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

Generally, the lamina axes (1, 2) do not coincide with the loading or global axes (x, y) (Fig. 2b). The modulus matrix is in the lamina coordinates and we have to transform it into global coordinates, so the transformation matrix [T] is specified as follows,

$$[T^{-1}] = \begin{bmatrix} m^2 & n^2 & -2mn \\ n^2 & m^2 & 2mn \\ mn & -mn & m^2 - n^2 \end{bmatrix}$$

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

Where $m = \cos \theta$ and $n = \sin \theta$

The lamina Global Modulus matrix is specified as

$$[Q]_{xys} = [T][Q]_{126}[T^{-1}]$$

$$\begin{bmatrix} N_x \\ N_y \\ N_s \end{bmatrix} = \sum_{k=1}^n \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \left\{ \int_{z_{k-1}}^{z_k} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{bmatrix} dz + \int_{z_{k-1}}^{z_k} \begin{bmatrix} kx \\ ky \\ ks \end{bmatrix} z^1 dz \right\}$$

$$\begin{bmatrix} M_x \\ M_y \\ M_s \end{bmatrix} = \sum_{k=1}^n \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \left\{ \int_{z_{k-1}}^{z_k} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_s \end{bmatrix} Z dz + \int_{z_{k-1}}^{z_k} \begin{bmatrix} kx \\ ky \\ ks \end{bmatrix} z^2 dz \right\}$$

$$\bar{Q}_{xx} = Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta$$

$$\bar{Q}_{yy} = Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta$$

$$\bar{Q}_{xy} = (Q_{11} + Q_{22} + 2Q_{12}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta)$$

$$\bar{Q}_{ss} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta)$$

$$\bar{Q}_{xs} = (Q_{11} - Q_{22} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{11} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta$$

$$\bar{Q}_{ys} = (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta$$

The same relationship is expressed in more compact form below

$$[\sigma]_k = [\bar{Q}]_k [\varepsilon^0] + Z_k [\bar{Q}]_k [k]$$

The above relations are expressed in terms of three laminate stiffness matrices [A], [B] and [D], which are the functions of the geometry, material properties and stacking sequence of the individual plies. These matrices are defined as follows, to combine the lamina stiffness it is necessary to invoke the definition of stress and moment resultant, N and M as integral of Stress through the thickness of the lamina. The overall stiffness properties of a composite lamina may now be expressed via the following matrix equation. Where the

A_{ij} , B_{ij} and D_{ij} are summation of lamina stiffness values, defined as shown.

$$A_{ij} = \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k - h_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^2 - h_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^n (\overline{Q}_{ij})_k (h_k^3 - h_{k-1}^3)$$

Where

A_{ij} Laminate extensional stiffness or in-plane laminate moduli

B_{ij} Laminate coupled bending-extension stiffness

D_{ij} Laminate bending or flexural stiffness

z_k Distance of individual layer k from the laminate reference

The overall load-deformation relations for this class of laminates are

$$[A^1] = [A^*] - [B^*][D^{*-1}][C^*] = [A^*] + [B^*][D^{*-1}][B^*]^T$$

$$[B^1] = [B^*][D^{*-1}]$$

$$[C^1] = [D^{*-1}][C^*] = [B^1]^T = [B^1]$$

$$[D^1] = [D^{*-1}]$$

$$\begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} = \begin{bmatrix} A^1 & B^1 \\ B^1 & D^1 \end{bmatrix} \begin{bmatrix} N \\ M \end{bmatrix}$$

Where ε^0_{xys} and κ_{xys} are the mid surface strains and curvatures. The plane stress constitutive equation is given by

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \varepsilon_{x0} \\ \varepsilon_{y0} \\ \gamma_{s0} \end{bmatrix} + z \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_s \end{bmatrix}$$

$$\{\sigma_{xys}\} = [Q_{xys}]\{\varepsilon_{xys}\}$$

Figure 3 shows the flow chart of analytical model that is used to develop the MATLAB program. The program takes the input of lamina thickness, orientation and material engineering constants and calculates the component stresses. These component stresses are then used for failure prediction using Tsai-Wu failure criteria.

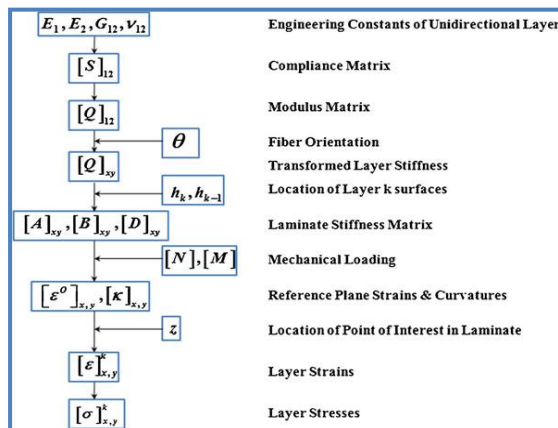


Fig. 3: Flow Chart for Mathematical Model

Finite Element Analyses of Composite Pressure Vessel:

The geometry of the pressure vessel under consideration is shown in Fig. 4. The pressure vessel with an outer diameter D 40 mm, length ‘ L ’ 230mm and thickness ‘ T ’ is loaded by internal pressure P 7.18 MPa and an external torque T 83kN-m and lamina thickness ‘ t ’ is 0.15mm. Material property of graphite/epoxy pressure vessel is shown in Table-1.

Table: 1. Properties of Graphite/Epoxy Laminate

Material properties of GRAPHITE/EPOXY LAMINATE	Values
Volume fraction (Vf)	60%
E1	88.53 Gpa
E2	6.72 Gpa
v12	0.28
v23	0.4
G12	4.03 Gpa
G23	1.022 Gpa
Tensile strength (Xt)	1560 Mpa
Transverse Tensile strength (Yt)	1760 Mpa
Compressive strength (Xc)	35.75 Mpa
Transverse Compressive strength (Yc)	178 Mpa
Shear Strength (S)	61.72Mpa

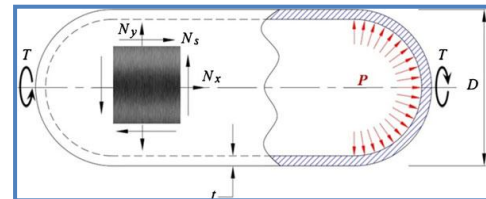


Fig. 4: Thin walled cylindrical pressure vessel under internal pressure and torque loading

Element type:

It is very necessary to select the appropriate element type for the accurate finite element analysis of the composite pressure vessel. The finite element software, ANSYS 14.5 provides the shell element 2 node 208 can be utilized to model layered composites layered structures and up to 8 uniform thickness layers can be modeled by this element. This model consists of 122 nodes, 60 elements

Analysis Criteria

The geometry and loading conditions on the pressure vessel are same as described above. 2 node linear layered structural shell element ‘‘SHELL208’’ has been used to create finite Element (FE) model of the structure. Full model of the vessel is modeled and boundary condition is applied at both two ends. The layer orientation of the vessel is selected as design variable from 0Deg to 90Degs, First Principal Stress is defined as state variable and the first layer principal stress of the pressure vessel is the objective function that has to be minimized. For above analysis we considered three layer orientations like $[0, \theta, \theta, 0]_s$, $[0_2, \theta_2]_s$, $[0, \theta_2, 0]_s$ finally we predict optimized ply orientation.

ANSYS Design optimization is a programmed mathematical technique that incorporates iterative design cycle into an automated process. The analysis, evaluation, and modification tasks are performed automatically, making it possible to obtain an “optimum” design more efficiently

Analytical Results and Discussion:

Case 1: $[0, \theta, \theta, 0]_s$

The Stress values and graph plots for $[0, \theta, \theta, 0]_s$ given below:

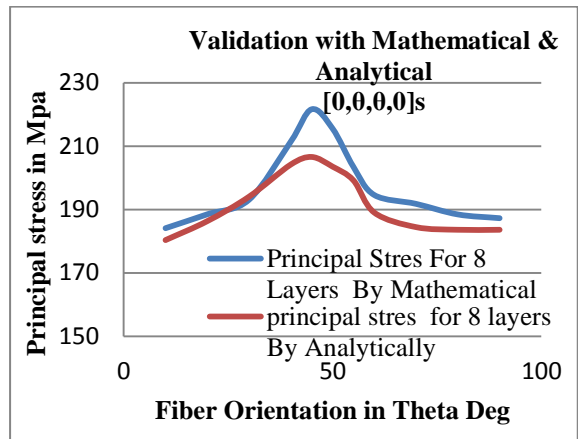
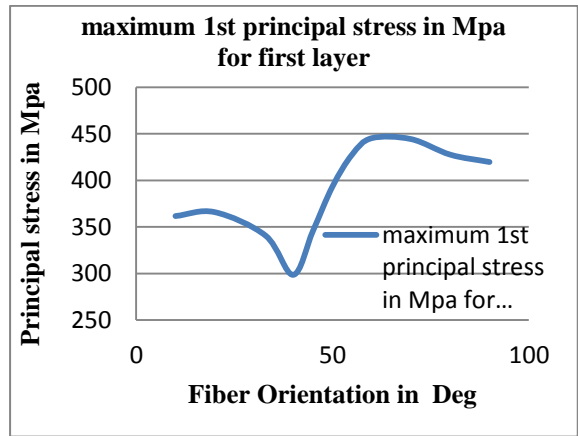
Table: 2.1: Tabular Format for Principal Stress at First Layer and Eight layers:

S.No	layer arrangement	maximum 1st principal stress in Mpa for first layer	1st principal stress for 8 layer
1	[0,10,0,10]	361.683	180.342
2	[0,20,0,20]	365.895	186.33
3	[0,30,0,30]	340.55	194.048
4	[0,40,0,40]	298.578	204.209
5	[0,45,0,45]	345.491	206.598
6	[0,50,0,50]	393.442	203.549
7	[0,55,0,55]	427.718	199.194
8	[0,60,0,60]	445.32	189.017
9	[0,70,0,70]	444.343	184.441
10	[0,80,0,80]	427.621	183.607
11	[0,90,0,90]	419.728	183.607

Table: 2.2. Tabular Format for Principal Stress For mathematical and analytical:

S. No	layer arrangement	Principal Stress For 8 Layers By Mathematical	1st principal stress for 8 layer Analytically
1	[0,10,0,10]	184.102	180.342
2	[0,20,0,20]	188.49	186.33
3	[0,30,0,30]	193.09	194.048
4	[0,40,0,40]	211.45	204.209
5	[0,45,0,45]	221.647	206.598
6	[0,50,0,50]	215.564	203.549
7	[0,55,0,55]	203.458	199.194
8	[0,60,0,60]	194.569	189.017
9	[0,70,0,70]	191.784	184.441
10	[0,80,0,80]	188.445	183.607
11	[0,90,0,90]	187.258	183.607

Graph for 1st Principal Stress for First Layer:



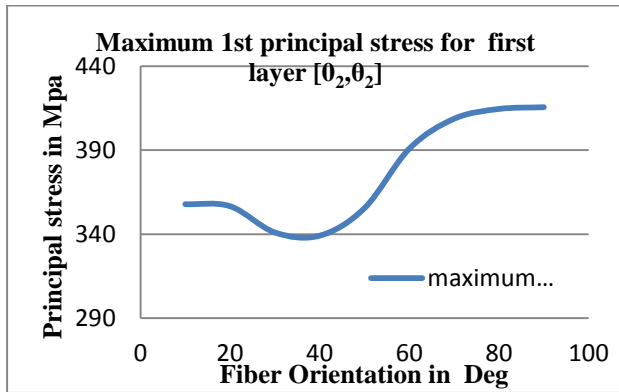
Case 2: $[0_2, \theta_2]_s$

The Stress values and graph plots for $[0_2, \theta_2]_s$ given below:

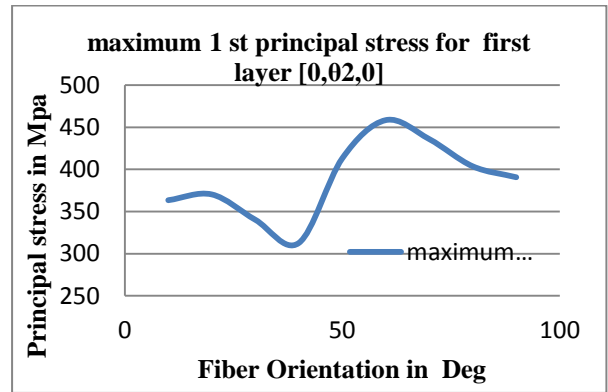
Table: 4.3. Tabular Format for Principal Stress at First Layer and Eight layers:

S. NO	layer arrangement	maximum 1st principal stress in Mpa for first layer	1st principal stress for 8 layer
1	[0,0,10,10]	357.832	178.829
2	[0,0,20,20]	356.63	181.756
3	[0,0,30,30]	340.958	185.672
4	[0,0,40,40]	339.091	189.88
5	[0,0,50,50]	355.5	193.771
6	[0,0,60,60]	390.906	196.756
7	[0,0,70,70]	408.716	198.564
8	[0,0,80,80]	414.524	199.346
9	[0,0,90,90]	415.556	199.529

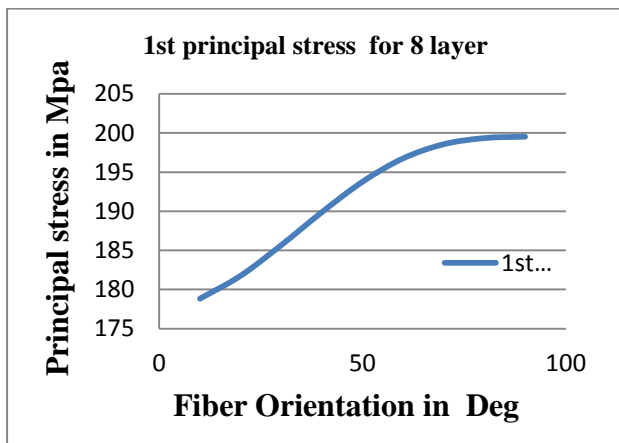
Graph for 1st Principal Stress for First Layer:



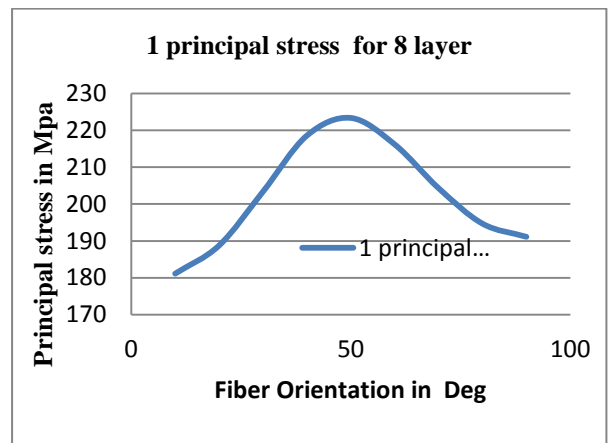
Graph for 1st Principal Stress for First Layer:



Graph for 1st Principal Stress for Eight Layers:



Graph for 1st Principal Stress for Eight Layers:



Case 3: $[0, \theta_2, 0]_s$

The Stress values and graph plots for $[0, \theta_2, 0]_s$ given below:

Table: 2.4. Tabular Format for Principal Stress at First Layer and Eight layers:

S. No	layer arrangement	maximum 1 st principal stress in Mpa for first layer	1 principal stress for 8 layer
1	[0,10,10,0]	363.424	181.121
2	[0,20,20,0]	370.28	188.752
3	[0,30,30,0]	340.274	203.372
4	[0,40,40,0]	312.589	218.579
5	[0,50,50,0]	413.05	223.372
6	[0,60,60,0]	458.428	216.194
7	[0,70,70,0]	435.925	204.316
8	[0,80,80,0]	403.622	194.694
9	[0,90,90,0]	390.629	191.117

Conclusion:

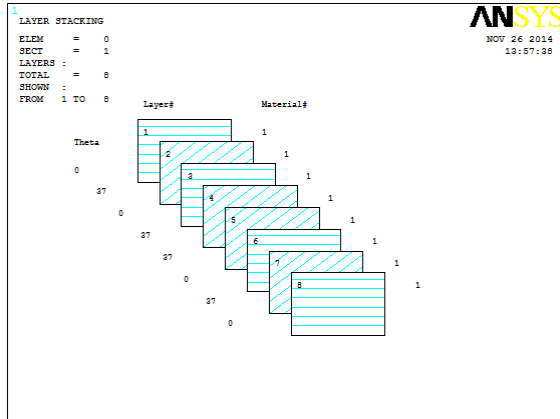
In the present scenario we consider the total number of layers are constant and maintain thickness value of 0.15mm for the composite pressure vessel. We can arrange the differential sequence as $[0, \theta, \theta, 0]_s$, $[0_2, \theta_2]_s$, $[0, \theta_2, 0]_s$. Consider the range of orientation angle between 0^0 to 90^0 . But the minimum value obtained at 0^0 degrees, so we consider first layer should be 0^0 degrees. We can found out the first principal failure calculations using ANSYS Classic (Numerical - Method) for the above differential sequence.

$$[0, \theta, 0, \theta]_s = [0, \theta, 0, \theta: \theta, 0, \theta, 0]$$

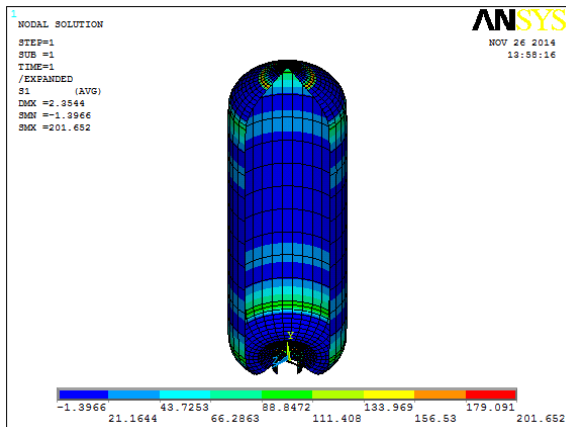
$$[0, 37, 0, 37]_s = [0, 37, 0, 37: 37, 0, 37, 0]$$

In this sequence we take 0^0 degree for the first layer is to maintain stress value minimum. In this total layers consider are eight and maintained thickness of 0.15mm. So we can found out the optimum angle for the 1st principal stress value minimum at 37^0 degrees and the stress value is **297.087** Mpa. And the optimum angle for the 1st principal stress value minimum at 37^0 degrees and the stress value is **297.03** Mpa this results are shown in below

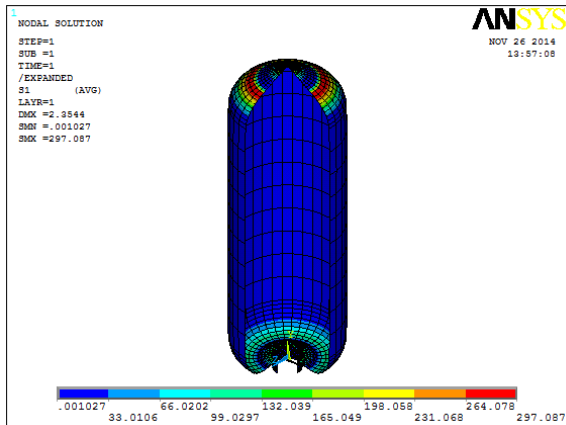
For optimized layer orientation results are like $[0, 37, 0, 37]_s = [0, 37, 0, 37: 37, 0, 37, 0]$



Layer arrangement orientation (0, 37, 0, 37) s



First principal stress for all layers (0, 37, 0, 37) s



First layer principal stress for (0, 37, 0, 37) s

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